Modelling and Control of PEMFC Based on Support Vector Machine

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***Abstract*—When current is drawn from a proton exchange membrane fuel cell (PEMFC), it is critical that the reacted oxygen is replenished rapidly by the air supply system to avoid oxygen starvation and damage. This paper proposes a support vector machine (SVM) model based model predictive control (MPC) strategy to maintain a necessary level of the oxygen excess ratio during abrupt changes in the stack current. Due to its excellent performance in function regression, SVM is used to establish PEMFC model by mapping PEMFC performance (for the oxygen excess ratio in this paper) as a function of various operation conditions. Based on the SVM model, a model predictive controller is designed using Model Predictive Control Toolbox. The SVM model and the model predictive controller have been implemented in the MATLAB/SIMULINK environment. Simulation results demonstrate the effectiveness of the model and the controller. The optimum oxygen excess ratio is able to be maintained during abrupt changes in the stack current.**

***Index Terms*—Air supply control; model predictive control; proton exchange membrane fuel cell; support vector machine.**

I. Introduction

Fuel cells are promising candidates for future power generation as they provide an efficient, clean, and reliable power solution. Over the last decade, there has been an ever increasing interest worldwide in proton exchange membrane fuel cell (PEMFC), due to its high power density, low operating temperature and fast start-up [1].

The PEMFC system without a proper controller will not be able to withstand the load fluctuations [2]. One of the most important challenges for the PEMFC controller is to ensure sufficient amount of oxygen provided in the cathode when fuel cell stack current abruptly changes due to the uncontrollable load. When a large load is applied to the fuel cell, the sudden increase in the current can cause the oxygen starvation if the depleted oxygen cannot be replenished immediately and sufficiently [3]. This catastrophic event permanently damages cells and limits the power response of the system. This problem arises from the fact that the oxygen reacts instantaneously as the current is drawn from the stack, while the air supply rate is limited by the manifold dynamics and the compressor operational constraints [4].

The control and management of air flow have been the focus in many publications. The oxygen excess ratio or stoichiometric ratio is defined as the ratio of inlet oxygen flow to reacted oxygen flow in the cathode. The oxygen excess ratio is indicative of oxygen starvation and can be considered as a good performance index. In [4]-[6], a dynamic PEMFC model is proposed and the control of the oxygen excess ratio is approached through the manipulation of the compressor motor voltage. In [4], an auxiliary power source, ultra-capacitor, is added to compensate the dynamics of fuel cell during the transient. The model predictive control strategy for air flow management in the hybrid fuel cell ultra-capacitor system is proposed. In [5], the static and dynamic feedforward controllers are designed, meanwhile in [6], the observer-based feedback controller is employed. In [7], a dynamic matrix control (DMC) based predictive control strategy is proposed for the control of fuel cell voltage and oxygen excess ratio. In [8], a multivariable model-based control scheme is proposed to control the oxygen excess ratio and the cathode pressure. In our recent work, an air flow control strategy based on the extreme seeking control algorithm is proposed to search for the optimum oxygen excess ratio online [9]. These researches reveal that PEMFC dynamical model is crucial for the control strategy.

This paper focuses on the design of the model predictive controller based on the support vector machine (SVM) model for the air flow control of the PEMFC. The model predictive control (MPC) strategy requires the system model to be as accurate as possible, while being simple enough to allow for repeated calculations during the receding horizon optimization [10]. However, most recent PEMFC models are theoretical (sometimes called "mechanistic") models, which are based on electrochemical, thermodynamic, and fluid dynamic relationships and using basic, phenomenological equations. Generally, they are computationally expensive and therefore difficult to apply to the control of fuel cells [11], [12]. In this paper, we present an attempt to build the SVM model of PEMFC, which provides low calculation burden for MPC. SVM is a statistics data-driven modelling approach. By mapping the oxygen excess ratio of PEMFC as a function of various operation conditions (for the stack current I_{st} and the compressor voltage V_{cm} in this paper), the SVM model can describe system behaviour without the knowledge of internal details. Using the Model Predictive Control Toolbox of the MATLAB program, the model predictive controller is then

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designed to maintain the optimum oxygen excess ratio during the transient following abrupt changes in the stack current.

The paper is organized as follows: In section II, the SVM theory is briefly explained and the SVM model for PEMFC is established. In section III, the model predictive controller is designed. Next in section IV, the system is simulated and results are discussed. Finally, the paper's main conclusions are provided in section V.

II. Support Vector Machine for PEMFC Modelling

The PEMFC model plays a vital role in the design of the control strategies. To meet the demands of developing efficient control strategies, efforts have been made to establish novel fuel cell models by the statistical data-driven approach. Based on statistical learning theory, support vector machine (SVM) is characterized by its simple topological structure and good generalization capability. SVM has been successfully applied in classification, function regression, and time series prediction, etc [13]. Due to its excellent performance in function regression [14], SVM can be used for nonlinear system identification and system control [15-17].

A. Theory of support vector machine

The basic idea of SVM is to map nonlinear inseparable input data into a high dimensional linear separable feature space via a nonlinear mapping technique (kernel dot product), and linear regression is done in the feature space. The major advantages of SVM are presented as follows [18]:

- By introducing the kernel, SVM avoids difficulties of using linear functions in the high dimensional feature space and optimization problem is transformed into dual convex quadratic programs.
- SVM delivers a unique solution, since the optimality problem is convex. This is an advantage compared to artificial neural networks, which have multiple solutions associated with local minima and for this reason may not be robust over different samples.
- SVM provides a good out-of-sample generalization. By choosing appropriate parameters, SVM can be robust, even when the training sample has some bias.

For clarity and completeness, the theory of SVM is briefly reviewed [19]:

For a given data set $\{(\mathbf{y}_k\ \mathbf{x}_k)|k=1,\ldots,n\} \subset \mathrm{R}^n \mathrm{X} \mathrm{R}$, $\mathbf{x}_k \in \mathrm{R}^n$ is the input data and $\mathbf{y}_k \in \mathrm{R}$ is the output data, n is the number of samples. The mission of SVM is to map the data set into a high dimensional feature space F via a nonlinear function Φ mapping and to do linear regression in this space:

$$y = (\omega \bullet \Phi(x)) + b \tag{1}$$

where $\Phi:\mathrm{R}^n \to F$, $\omega \in F$, b is threshold.

Since Φ is fixed, ω is determined from the data by minimizing the sum of the empirical risk functional $R_{\mathrm{emp}}[f]$ and a complexity term $\|\omega\|^2$, which enforces flatness in the feature space:

$$R_{\mathrm{reg}}[f] = R_{\mathrm{emp}}[f] + \lambda\|\omega\|^2 = \sum_{i=1}^{n} C(f(x_i) - y_i) + \lambda\|\omega\|^2 \tag{2}$$

where n denotes the sample size, $C(\cdot)$ is a cost function determining how we penalize estimation errors and λ is a regularization constant.

Equation (2) can be minimized by solving a quadratic programming problem, which is uniquely solvable. The vector ω can be written in terms of the data points:

$$\omega = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*)\Phi(x_i) \tag{3}$$

with α_i, α_i^* being the solution of minizing $R_{\mathrm{reg}}[f]$.

Taking (1) and (3) into account, the regression function is able to be acquired in the low dimensional input space:

$$f(x) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*)(\Phi(x) \bullet \Phi(x_i)) + b = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) K(x, x_i) + b \tag{4}$$

where $K(x, x_i)$ is the kernel function.

In (4) the kernel function is introduced to address the problem of dimensionality. It is defined as a function that corresponds to the dot product of two vectors in feature space:

$$K(x, x_i) = \Phi(x) \bullet \Phi(x_i) \tag{5}$$

It can be shown that any symmetric kernel function K satisfying Mercer's condition corresponds to a dot product in some feature space. A commonly used kernel is radial basis function (RBF) kernel: $K(x,x_i) = \exp\{-| x-x_i |^2/2\sigma^2\}$, which is also chosen in this paper.

In (2), a widely used cost function is ε-insensitive loss function, which describes how the estimated function $f(x)$ deviates from the actual one:

$$C(f(x) - y) = \begin{cases} |f(x) - y| - \varepsilon & \mathit{for}\ |f(x) - y| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where $\varepsilon > 0$ is a predefined constant.

B. Modelling PEMFC by support vector machine

Assume the PEMFC system can be described as:

$$\begin{aligned} y(k+1) = f[&y(k), y(k-1), \ldots, y(k-n_y), \\ &x(k), x(k-1), \ldots, x(k-n_x)] \end{aligned} \tag{7}$$

where x and y are the input and output of the system, n_x and n_y denote the input and output order, nonlinear function f is unknown.

If we let:

$$u(i) = (y(i), y(i-1), \ldots, y(i-n_y), x(i), x(i-1), \ldots, x(i-n_x)) \tag{8}$$

Then we can get:

$$y(i+1) = f[u(i)] \tag{9}$$

So the training data is $\{u(i), y(i+1)\}$.

The output can be obtained by using SVM to map the data set to high dimensional space:

$$y(k+1) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) K(u(i), u(k)) + b \tag{10}$$

To build the SVM model, the SVM learns from training data to define the mathematical relationship between the system inputs and outputs. Then the SVM model is validated with testing data to verify its performance. In order to compare with traditional modelling approach, training data and testing data for SVM modelling are acquired from the PEMFC model developed by J. T. Pukrushpan et al. in the MATLAB/SIMULINK environment [4]-[6]. However, it is worth pointing out that one can also get the training and testing data from experiments of real PEMFC. In this way, the SVM model can achieve better performance and enhance its practical value.

Fig. 1 shows the schematic of the PEMFC system and the parameters used in this model are given in Table 1. Most parameters are based on the 75 kW stacks used in the FORD P2000 fuel cell prototype vehicle [20]. This model takes both the fuel cell stack and the auxiliary systems into account. And the internal details are studied in depth. Electrochemical relations, flow equations and thermodynamic principles are used to determine the internal states of the PEMFC.

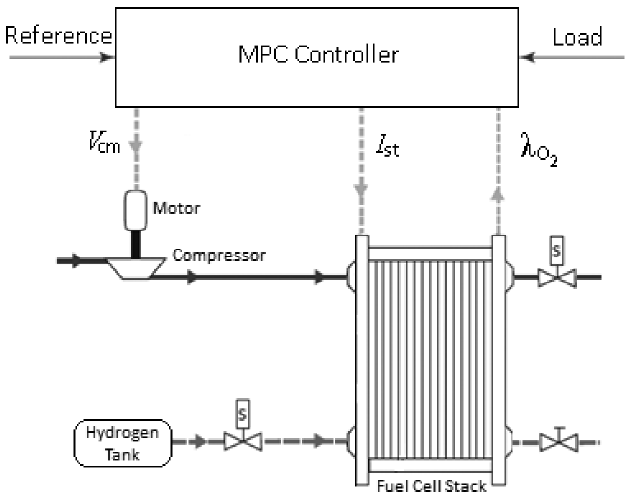


Fig. 1. Schematic of the PEMFC system and the controller

Table 1 Fuel Cell System Model Specifications

Fuel Cell Stack	No. of Cells (n)	381
	Maximum Power	75kW
	Cell Active Area	280cm^2
Air Compressor	Type	Centrifugal
	Maximum Power	12.5kW
	Diameter	0.2286m

In this research, the oxygen excess ratio λ_{O2} is maintained by adjusting the compressor voltage V_{cm} during the transient following abrupt changes in the stack current I_{st}. Therefore, to build the SVM model, the input $x(k)$ is set as $x(k)=\{I_{st}(k), V_{cm}(k)\}$, and and the output $y(k)$ is $\boldsymbol{y}(k)=\{\lambda_{O2}(k)\}$. Among them, V_{cm} is the control input and I_{st} is the disturbance input, which is the result of the uncontrollable load change. Compared with the MATLAB model, the SVM model maps the output as a function of the inputs by regression, disregarding the complex internal details.

Several points are worth noting in the preparation of training data and testing data. The training data ought to cover the entire expected operation range of the SVM model. In most cases, training data and testing data should be scaled, normally linearly. Scaling can increase training speed and result accuracy. Besides, training data and testing data must be scaled to the same interval. In this paper, the operation ranges of I_{st} and V_{cm} for SVM model are [100 A, 300 A] and [100V, 300V], both scaling to [0, 1]. LIBSVM 3.0, a powerful MATLAB toolbox for SVM classification and regression, is employed to establish PEMFC model in this paper [21].

In order to verify the performance of the proposed SVM model, we generate a time evolution of the stack current and the compressor voltage that act as the input to the MATLAB model. The corresponding output is recorded as the actual output in Fig. 2(a). Then the same time evolution is input to the SVM model and the corresponding output is depicted as the predicted output in Fig. 2(a). The predicted error, which is defined as the difference between the actual value and the predicted value, is given in Fig. 2(b). As can be seen from Fig.2, the predicted results are in good agreement with the actual values, and the maximal predicted error is not beyond 0.25.

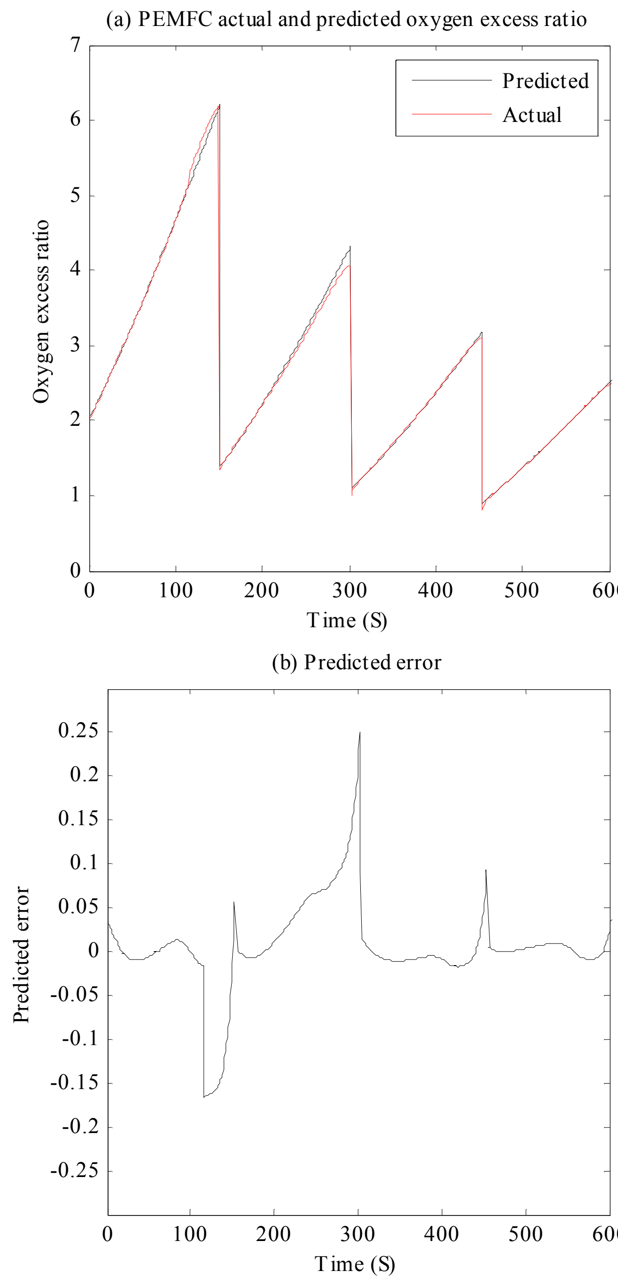


Fig. 2. Predicted oxygen excess ratio and error by SVM model

III. Model Predictive controller design

The design of the model predictive controller in this section is carried out using the Model Predictive Control Toolbox of the MATLAB program [22]. The Model Predictive Control Toolbox is a powerful tool for the design, analysis, and implementation of the model predictive control algorithm. It provides a convenient graphical user interface (GUI) for the model predictive controller design. The core MPC Toolbox algorithm is based on a model of the system to be controlled, a performance index driving the selection of the decision variables.

The schematic of the proposed controller and the SVM model developed in the MATLAB/SIMULINK environment is shown in Fig. 3. When there is an MPC block, the natural choice is to associate PEMFC input, i.e. the compressor voltage, with the MPC block output port and PEMFC outputs, i.e. the oxygen excess ratio, with the MPC block's "measured outputs" port.

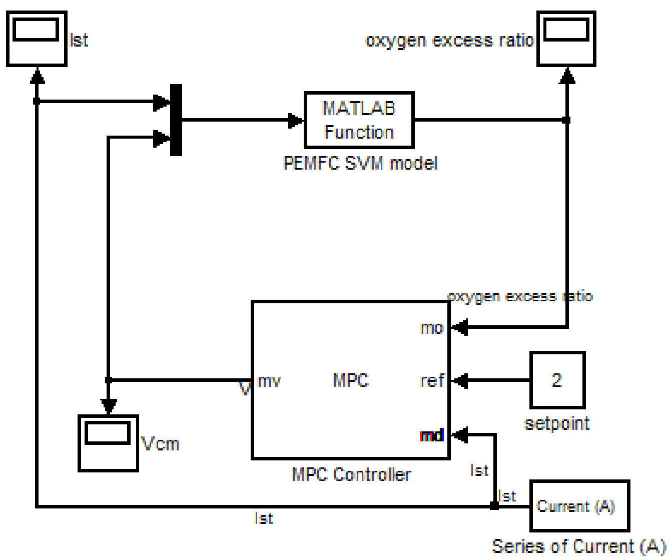


Fig. 3. PEMFC control system implemented in the SIMULINK

The model predictive controller comprises an internal model for the computation of the control movements. The MPC toolbox requires the model used in controller design to be affine, i.e., a linear, time-invariant (LTI) system describing deviations from a nominal condition. One can define such a model by creating a state space model or linearizing a SIMULINK model. In this paper, we create the state space model by using linearized system matrices provided in [6].

Once the internal plant model has been defined, the remaining design decisions comprise:

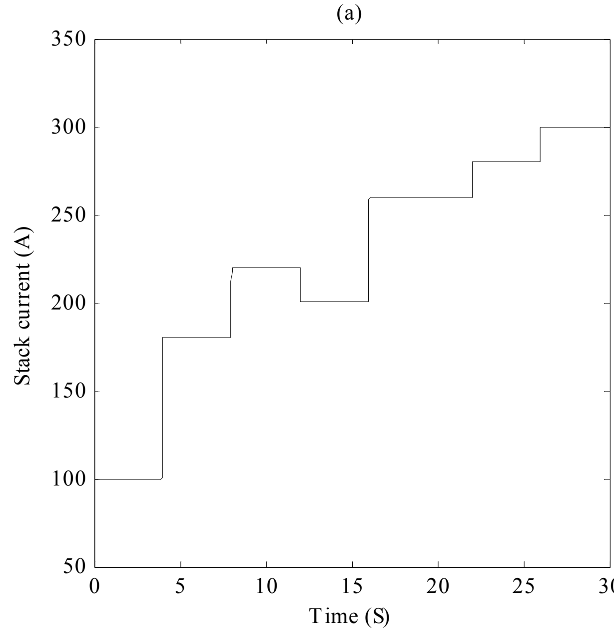
- Specifying signal properties and assigning their nominal values. In this paper, the manipulated variable is the compressor voltage V_{cm}, the measured disturbance is the stack current I_{st}, the output is the oxygen excess ratio λ_{O2}. The nominal value for the oxygen excess ratio is 2.
- Specifying controller properties. The selection of prediction horizon P and control horizon M is a tuning process. The designers usually choose P and M such that controller performance is insensitive to small adjustments in these horizons [22]. In this paper, the prediction horizon and the control horizon are both set to 3 samples. The control interval is set as 0.1 time unit.
- Specifying constraints. The oxygen excess ratio and compressor voltage have lower bound, 1 and 0, respectively.
- Weight tuning. The weights specify the trade-off between robustness and response speed. The weight for output is set to 1.0, and the weight for manipulated variable is set to 0.1.

IV. Simulation and result

In this section, the results of simulation of the transient response of the PEMFC system are presented and discussed. For the purpose of comparison, the objective of the proposed MPC strategy is the same as Pukrsphan's, that is to maintain the optimum oxygen excess ratio $\lambda_{O2} = 2$ during the transient following abrupt changes in I_{st} [6]. Besides, the time evolution of the stack current that acts as an input disturbance to the system is shown in Fig. 4(a). Also, the time evolution of the stack current is set as the same one provided in [6]. The time evolution of the performance variable, the oxygen excess ratio, is shown in Fig. 4(b). The corresponding variation in the controlled input, the compressor motor voltage, is shown in Fig. 4(c).

The results displayed in Fig. 4(b) show that the proposed controller performs quite well with respect to maintaining the oxygen excess ratio at the optimal value consistent with a given input level of the stack current. Besides, no overshoot is observed in Fig. 4(b). This is favorable as no redundant power is used to produce the unnecessary overshoot. However, the proposed controller suffers from some deficiencies. Its response speed is relatively slow, which leads to long settling times as shown in Fig. 4(b).

(a)



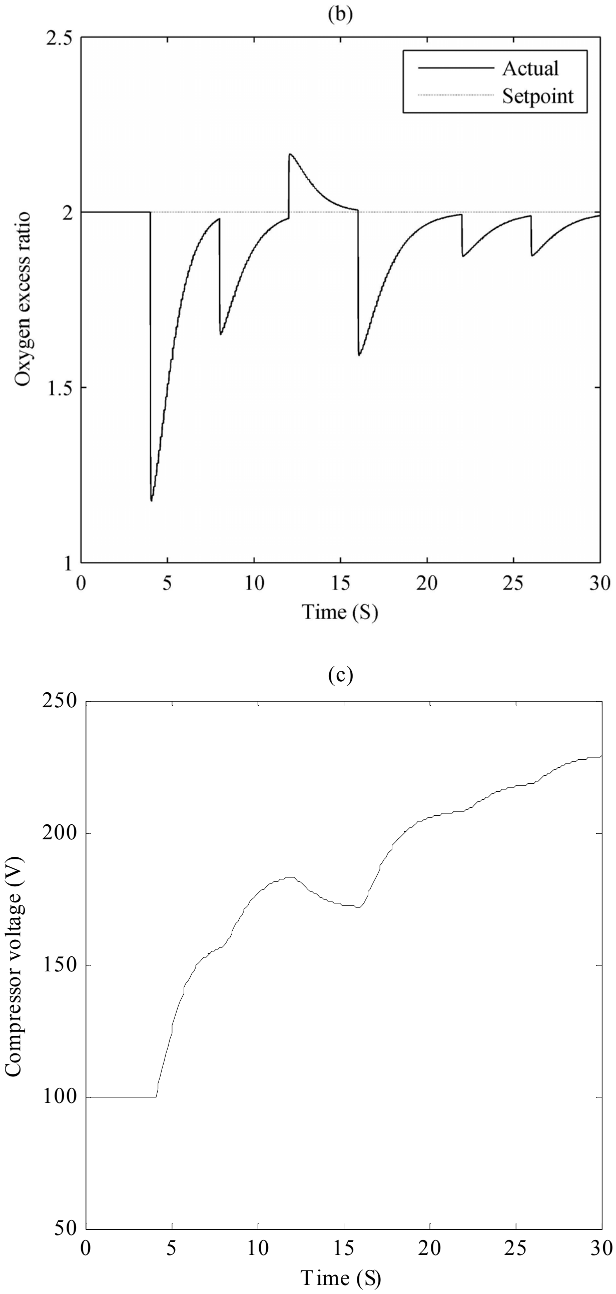


Fig. 4 Response of PEMFC to step-like temporal variation of the stack current

V. Conclusion

In this paper, the model predictive controller for PEMFC is designed based on the SVM model. First, the model of PEMFC system is established through using SVM. By mapping oxygen excess ratio of PEMFC as a function of various operation conditions (for the stack current I_{st} and the compressor voltage V_{cm} in this paper), the model avoids the internal complexity of PEMFC and thus provides low computational burden for the control algorithm of PEMFC. Simulation results demonstrate the accuracy of the SVM model.

The model predictive controller is designed by using Model Predictive Control Toolbox in the MATLAB/SIMULINK environment. The control system is then simulated to verify the performance. Simulation results demonstrate the effectiveness of the proposed control strategy. The optimum oxygen excess ratio is able to be maintained during the transient following abrupt changes in I_{st}. However, the proposed controller has some drawbacks in terms of response speed.

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VI. Biographies

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