

# Modelling Power Transformers at Power Line Carrier Frequencies

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## Abstract

Many electricity suppliers will be installing smart meters during the next few years. It is likely that these smart meters communicate with the electricity suppliers' control centre using power line carrier (PLC) communications. The CENELEC A frequency band from 9 kHz to 95 kHz is set aside for use by Power Supply Companies. The upper part of this frequency band is very suitable for smart meter reading and smart grid control.

In most practical situations, the PLC signals are injected directly onto the LV lines. However it may be desirable to transmit the PLC signals through the MV network and pass them through the LV distribution transformer, as is done in the present ripple control signalling. To design a suitable communication system using this technique, the PLC signal attenuation through the transformer needs to be known. To determine these losses, it is necessary to know how LV distribution transformers behave at these PLC frequencies and what isolation they provide between the LV and the HV distribution networks.

ERGON is an Australian Electricity supplier operating in rural Queensland. They have sponsored a project carried out by James Cook University, to develop a communication system for SWER lines, so that smart metering, fault detection and enhanced network control can be applied to SWER lines. In this application the PLC signals are carried on the 19.1 kV SWER lines and the SWER line transformer becomes an integral part of the coupling network, for coupling the communication signals onto the SWER line. As part of this work a high frequency model of a 10 kVA and a 25 kVA power transformer was developed.

The paper describes how the transformer parameters of these transformers are measured and shows how those measurements are used to develop the transformer model covering frequencies from 50Hz to 1MHz.

## Introduction

In Queensland, Australia, ripple control is used, where audio frequencies are transmitted on the power network, to turn water heaters and air conditioners on and off in order to minimise the peak loads. During the last few years, the low frequency noise on the power lines has been increasing due to the increasing use of power electronic devices, in particular Compact Fluorescent Lamps (CFL) and inverter air conditioners. As a result the power levels required for the ripple control systems have had to be increased to maintain adequate control.

For Smart Grid applications a communication system is required between the control room of the electricity retailer and the customer, so that non-essential devices can be turned off during peak loads. One option for such a communication system is to use control signals in the CENELEC A frequency band from 9 kHz to 95 kHz, which is set aside for use by Power Supply Companies.

Since users are supplied through a LV transformer, one needs to know the behaviour of LV transformers, since they will have a significant effect on the PLC signals. If the PLC signals are to be generated at MV or HV substations, then the PLC signals must pass through the LV transformers, just like the ripple control signals do at present. If the PLC signals are coupled directly onto the LV lines, then the impedance of the transformer may be such that most of the PLC signals are lost in the transformer and do not reach the consumer.

JCU has been working on a project with ERGON to develop a PLC communication system for use on SWER lines. To couple signals onto the 19.1 kV SWER line a low cost coupling network is required. The characteristics of the SWER line transformer at the source and customer ends of the line are critical in the efficiency of the coupling networks. As a result a model for the SWER transformer at PLC frequencies was developed. That same model with slightly different parameters will apply to LV distribution transformers as well as HV transformers.

## Development of a transformer model

In a practical transformer, the 240 V low voltage (LV) winding and the high voltage (HV) windings each have two terminals. For a single phase transformer the earth side of the windings is connected to the transformer case and earth, as shown in figures 1 and 2.

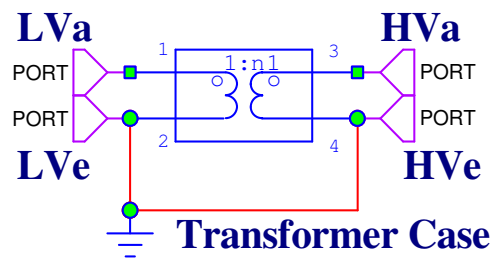


Figure 1. Ideal transformer model.

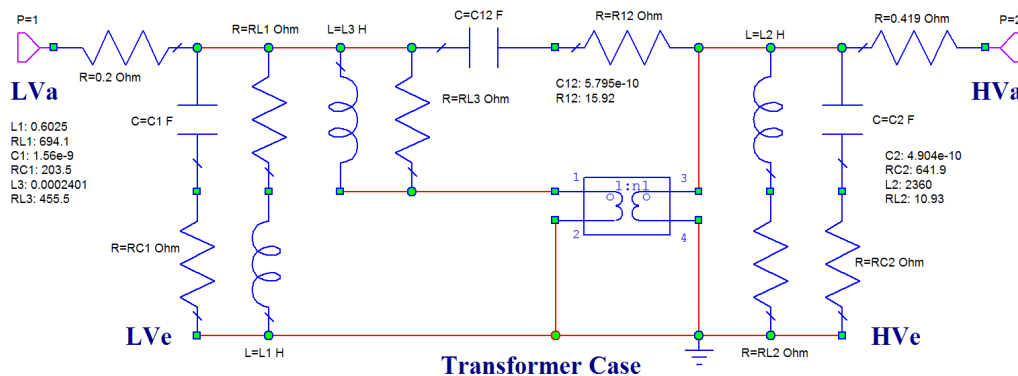


Figure 2. Wide frequency range transformer model.

The high frequency model of the transformer is shown in figure 2. Joan [1] uses a similar model, but does not produce any frequency responses. Other researchers [2-5] produce transformer models which do not yield a simple schematic circuit and only apply to circuit transients. The frequency response of the magnetic core is such that the magnetic coupling ceases to function at high frequencies [1]. The magnetic coupling is included in the model as an ideal transformer, with an 80 turns-ratio to convert the 240 V low voltage (LV) to the 19.1 kV high voltage (HV) of the SWER line. 19.1 kV is the phase to Neutral voltage of a

33 kV MV distribution system. The decrease in magnetic coupling with frequency is modelled by including an inductor, L3, in series with this ideal transformer. It was found a better impedance match was obtained by including a resistor RL3 in parallel with this inductor L3. At low frequency, the LV and HV windings reflect the magnetising inductance of the transformer, which is modelled as an inductor, L1 or L2, in series with resistors, RL1 or RL2, in parallel with the ideal transformer LV and HV terminals respectively.

At high frequencies the LV and HV windings each have a capacitance associated with them, due to capacitive coupling between the coils making up the winding. This is modelled by a capacitor, C1 or C2, in series with a resistor, RC1 or RC2, representing the losses of these high frequency winding impedances. In addition, there is a capacitive coupling between the LV and HV windings, which is modelled by a capacitor, C12, in series with a resistor, R12, representing the losses associated with this coupling. Normally a lossy capacitor is modelled using a parallel RC network. It was however found that a series RC network gave a better match.

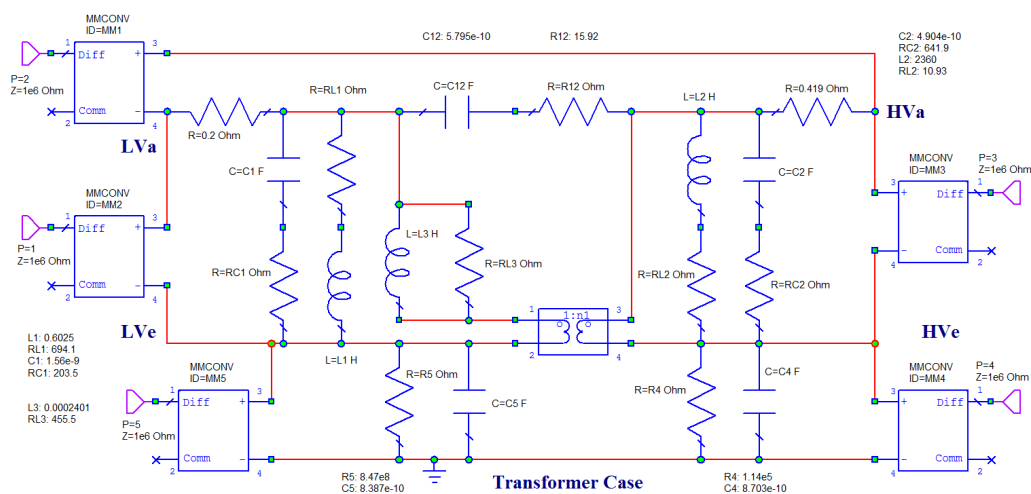


Figure 3. Transformer measurement points

A practical transformer has completely isolated LV and HV windings. The terminals for the LV winding can be labelled LV Active (LVA) for the hot side of the LV winding and LVEarth (LVE) for the side of the LV winding that is normally connected to the case or earth. Similarly the HV windings has HVA and HVE terminals. The case is normally connected to earth and to the LVE and HVE winding terminals.

To develop the PLC frequency model of the transformer, it is desirable to do impedance measurements of the LV (LVA to LVE) winding, the HV (HVA to HVE) winding, coupling between the LV and HV windings (LVA to HVA), the capacitive coupling between the LV winding and the case (LVE to Case) and the capacitive coupling between the HV winding and the case (HVE to Case). For a completely accurate model, additional impedance measurements can be made, like HVA to Case, LVA to Case, HVA to LVE and LVA to LVE. As is shown in this paper, a sufficiently accurate transformer model can be made using the 5 measurements shown in figure 3.

The problem now becomes how to match the components in the model to the impedance measurements. One could write down equations for the impedances that can be seen in each of the 5 measurement points and solve the resulting complex equations to obtain the component values. Since the impedances of the inductances and capacitances change with frequency, solving these equations is an exceedingly difficult task.

It is possible to use modern circuit simulation and optimisation tools, to "solve" the simultaneous equations by optimisation and match the circuit elements to the measurements. To illustrate this technique, the process is performed on a relatively simple network.

## Bandpass T Matching Network

A simple Bandpass T matching network [6], which in RF electronics is used to match power transistors to  $50 \Omega$  loads, is used to illustrate the impedance matching process. The Bandpass T network and all the possible measurement ports are shown in figure 4. For this example, a network is chosen to match a device with  $X_D = 5 + j 0.9 \Omega$  to a load  $X_L = 50 \Omega$  at 150 MHz with a Q of 3.5. The resulting component values are calculated [6] to be:  $L = 17.61 \text{ nH}$ ,  $C1 = 46.92 \text{ pF}$  and  $C2 = 37.22 \text{ pF}$ .

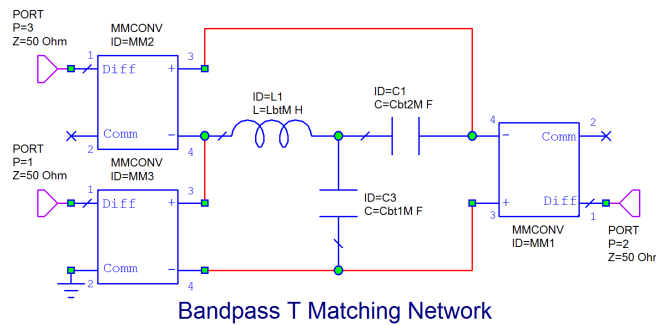


Figure 4. Bandpass T matching network.

!	Port1		Port3		Port2	
!(Hz)	Ohm	Rad	Ohm	Rad	Ohm	Rad
!Freq	Z(1,1)	Ang(Z(1,1))	Z(1,1)	Ang(Z(1,1))	Z(1,1)	Ang(Z(1,1))
50e6	62.3110	-1.55762	79.9906	-1.55906	153.359	-1.56413
60e6	49.8983	-1.55711	64.6312	-1.55896	127.800	-1.56524
70e6	40.7159	-1.55607	53.3440	-1.55843	109.543	-1.56603
80e6	33.5526	-1.55454	44.6020	-1.55753	95.8505	-1.56663
90e6	27.7354	-1.55246	37.5569	-1.55629	85.2006	-1.56709
100e6	22.8605	-1.54972	31.6996	-1.55466	76.6807	-1.56746
110e6	18.6711	-1.54604	26.7062	-1.55257	69.7098	-1.56777
120e6	14.9959	-1.54096	22.3608	-1.54987	63.9007	-1.56802
130e6	11.7165	-1.5336	18.5140	-1.54632	58.9853	-1.56823
140e6	8.74862	-1.52203	15.0590	-1.54147	54.7721	-1.56842
150e6	6.03109	-1.50125	11.9177	-1.53453	51.1206	-1.56857
160e6	3.52102	-1.45323	9.03174	-1.52378	47.9256	-1.56871
170e6	1.21379	-1.22803	6.35699	-1.50497	45.1064	-1.56884
180e6	1.14530	1.21055	3.86149	-1.46369	42.6005	-1.56894
190e6	3.19523	1.44523	1.53779	-1.30206	40.3584	-1.56904
200e6	5.18441	1.49413	0.851349	1.07574	38.3405	-1.56913
210e6	7.09425	1.51516	2.90157	1.43210	36.5148	-1.56921
220e6	8.93256	1.52688	4.92187	1.48979	34.8550	-1.56928
230e6	10.7079	1.53435	6.86867	1.51315	33.3396	-1.56935
240e6	12.4278	1.53953	8.74723	1.52579	31.9504	-1.56941
250e6	14.0989	1.54335	10.5648	1.53372	30.6724	-1.56946

Table 1 Bandpass T measured impedances

The impedances measured at the 3 ports are shown in table 1. For simplicity these measurements have been produced using the circuit simulation software for a circuit with the component values calculated as above. Three networks are then produced, each with one measurement port, the one for port 1 is shown in figure 5.

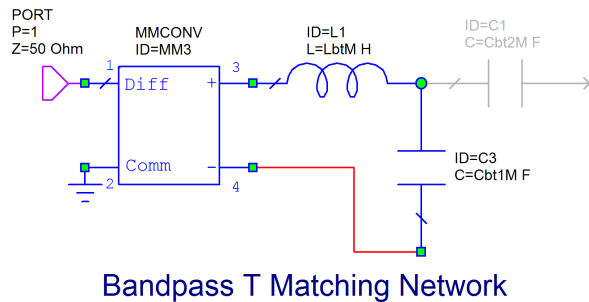


Figure 5. Network for matching port 1 measurements.

Error equations, which correspond to a normalised difference between the measured impedances and those obtained from the impedance seen at port 1 in figure 5 are produced. For the network of figure 5, those equations are:

$$EP1Z11M = (MeP11Z11M - MoP11Z11M) / (MeP11Z11M + MoP11Z11M) \quad (1)$$

$$EP1Z11P = (MeP11Z11P - MoP11Z11P) \quad (2)$$

Where EP1Z11M is the error for the magnitude of Z11 for Port 1, MeP11Z11M is the measured value of the magnitude of Z11 for Port 1 and MoP11Z11M is the magnitude of Z11 for Port 1 of the model.

EP1Z11P is the error for the phase of Z11 for Port 1, MeP11Z11P is the measured value of the phase of Z11 of Port 1 and MoP11Z11P is the phase of Z11 for Port 1 of the model.

Initial guesses for the values of the components of the network are then made and the circuit simulator is then set to optimise the guessed component values to minimise the error equations (1) and (2), and similar equations for the other ports of figure 4.

All the errors reduce to zero and this optimisation then produces exactly the same values that were calculated, thus demonstrating that this technique can be used for matching complex networks to measured impedances.

## 10kVA and 25 kVA Measurements

The impedances of three 10 kVA SWER line transformers and a 25 kVA SWER line transformer were measured over a wide range of frequencies. Impedance measurements were made between LVA and LVE terminals, between the HVA and the HVE terminals, between the LVA and the HVA terminals, between the LVE terminal and the transformer case and between the HVE terminals and the transformer case. These measurements were compared to determine if a common model could represent them. Figures 6 and 7 show the measurements for the LV winding and uniquely identify each transformer's measurement by its serial number.

As shown in figures 6 and 7, the four transformers have similar impedances over a wide frequency range, with the three 10kVA transformers grouped together and the 25kVA transformer somewhat apart. This suggests that a single 10kVA model and a similar 25kVA model could be used as a suitable model for the transformers.

Figures 6 show some differences between transformers in the 10 kHz to 100 kHz frequency range. The measurements for figures 6 and 7 were done at a reasonably coarse frequency interval. As a result, the measurements for one 10 kVA transformer were repeated at 10 steps per decade and used for the optimisation presented in this paper. Those measurements also show such resonances. This implies that for part of the CENELEC A frequency band, one can expect significant variations in the impedance looking into the LV windings. The impedance is also less than 100  $\Omega$  for this frequency range so that the LV winding may present a significant load for PLC coupling networks.

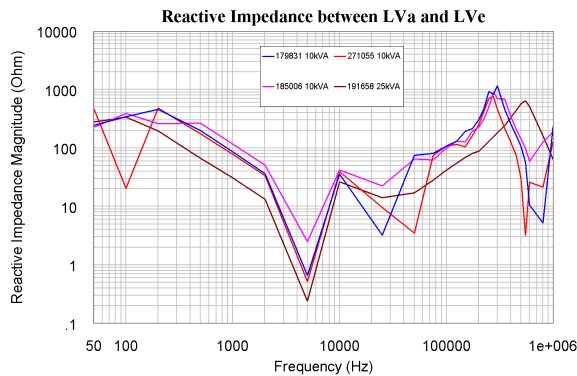


Figure 6. LV winding impedances.

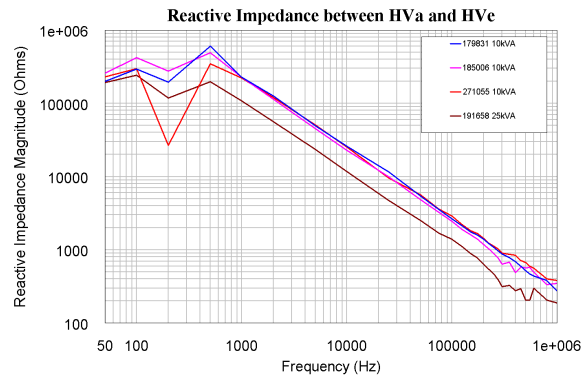


Figure 7. HV winding impedances.

Initially error equations, like (1) were made for both Real and Reactive parts of the impedances. The optimisation resulted in a good match for the HVA to HVE, the HVA to LVA, the LVE to Case and the HVE to case measurements. It was however difficult to obtain a good match for the LVA to LVE windings, since firstly both capacitive and inductive regions exist as shown in figure 6 and secondly the impedances are much smaller than those of the HVA to HVE and HVA to LVA impedance.

## Transformer Model

It was found that a better match could be obtained if the normalised difference of magnitudes of the impedances and the difference between the phase of the measured and modelled impedances were minimised by optimisation, as shown in equation (1) and (2). Since the magnitude of the impedances is more important than the phase angle, they were given a larger weighting in the optimisation. Since normally LVE and HVE are connected to the transformer case, the LVE-Case and HVE-Case measurements were also given less weight. Since the PLC signals are less than 200kHz, no optimisation was done for the frequencies above 1 MHz. The component values resulting from the optimisation are shown in figures 2 and 3 and a comparison between the measured and modelled impedances are shown in figures 8 to 12

For the LVA-LVE measurements shown in figure 8, the measurements and model give a reasonable match for the 100 Hz to 4 kHz frequency range and a better match for the 40kHz to 200kHz frequency range.

The falling impedance with frequency from 100 Hz to 2 kHz is due to the magnetic coupling as well as the other impedances in the circuit. The HV winding impedances are scaled by the inverse of the turns ratio squared as they are reflected into the LV side of the transformer. The rising impedance from 10 kHz to 100 kHz, is due to the inductor L1. The falling response for frequencies above 200 kHz is due to the LV winding capacitance C1.

Figure 8 shows that from 4kHz to 30 kHz the impedances are very small. Under these conditions, even small measurement errors, such as can occur due to the measurement lead resistances not being compensated for, will give significant percentage errors. As a result the weight of the error function for the LV winding at small impedance values is reduced.

Since our impedance bridge has a poor accuracy above 100 kΩ, no results are plotted and no optimisation was done for measurements and frequency ranges where the impedances were larger than 1 MΩ. This bridge limitation also results in phase differences between the measurements and model for those frequencies.

There is a very good agreement between the measurements and the model for all the other measurements, shown in figures 9 to 12. For all these measurements capacitive effects dominate. To obtain the best match at higher frequencies, a resistor in series rather than in parallel with the capacitors C1, C2 and C12, was required.

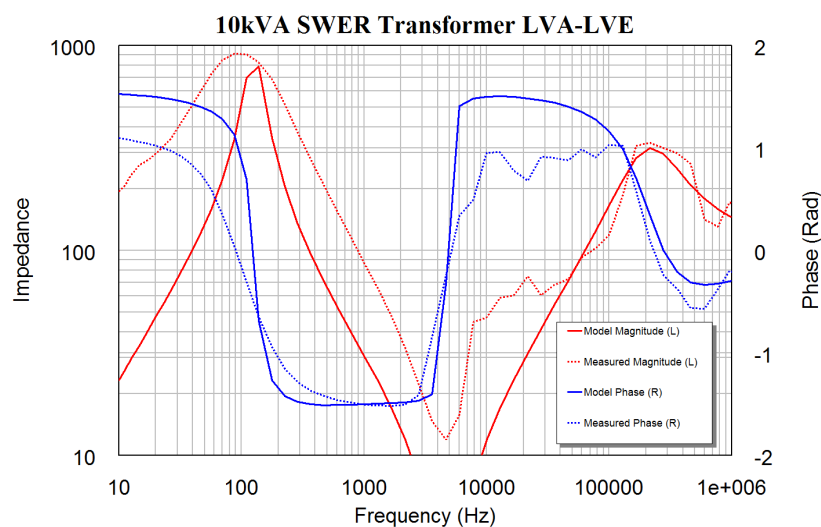


Figure 8. LV winding impedance.

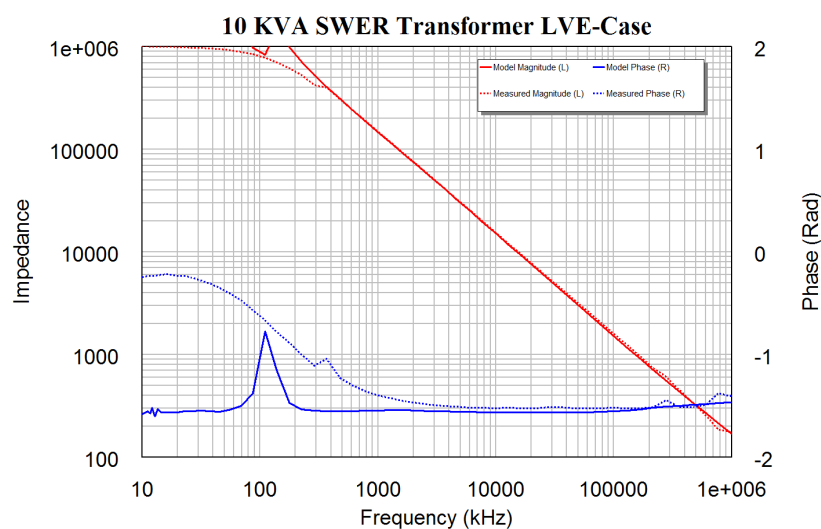


Figure 9. LVE-Case impedance

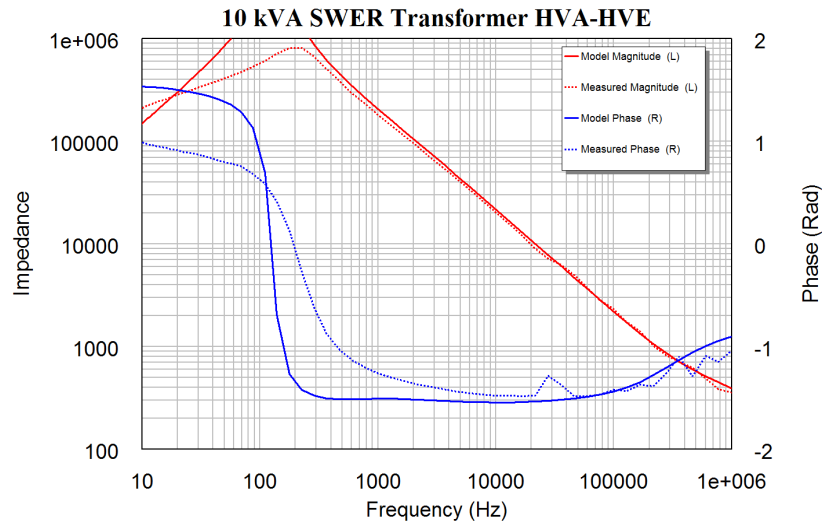


Figure 10. HV winding impedance.

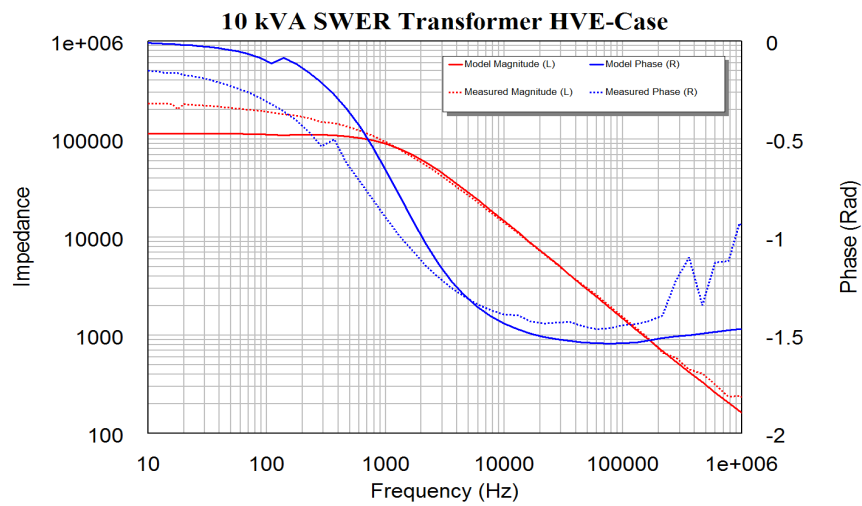


Figure 11. HVE-Case Impedance

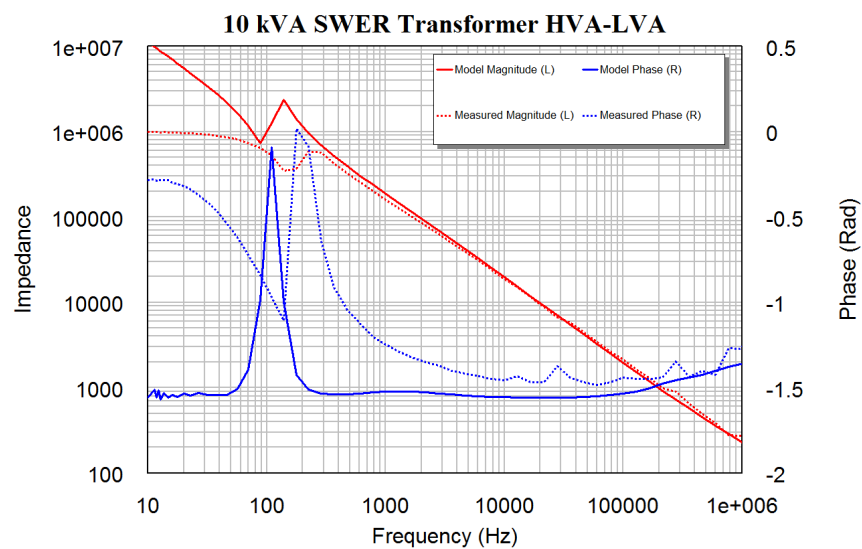


Figure 12. HVA-LVA Impedance.



The model, can be improved for frequencies below 1kHz, there the magnetic coupling dominates. This is mainly due to firstly the  $< 1 \text{ M}\Omega$  accuracy limitation of our impedance bridge and secondly due to a simple magnetic model being used. However since this model is designed to represent the power transformer at PLC frequencies, the model is very suitable for inclusion in the design of PLC systems.

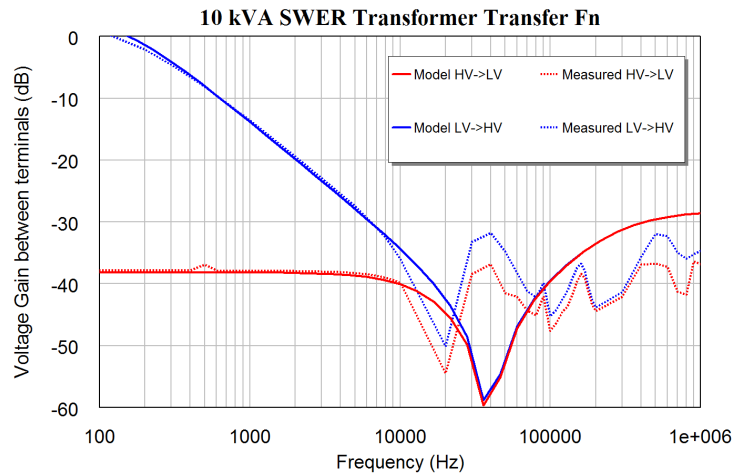


Figure 13. Voltage Transfer Function.

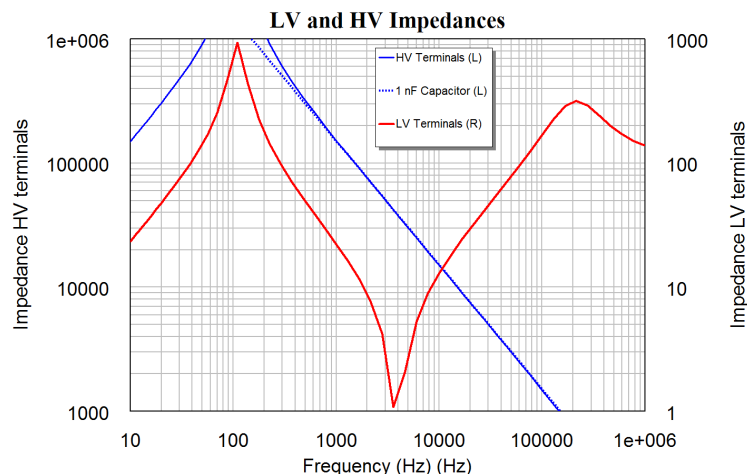


Figure 13. SWER Transformer Transfer Function.

Figures 13 and 14 show the performance of the transformer in the normal connection, with HVE, LVE and the Case connected to earth. Figure 13 shows the voltage transfer function, with the ratio of the terminal voltages plotted when a signal generator is connected to one end of the transformer and two  $50 \Omega$  terminated instruments, a digital waveform recorder and a spectrum analyser are both connected to the other winding of the transformer. The same condition is implemented in the model. There is a very good agreement between the measurements and the model for frequencies below 20 kHz and a reasonable agreement for other frequencies. The results in the CENELEC A band show the biggest discrepancies. The 40 dB to 60 dB loss show that it is undesirable to use the transformer to pass PLC signals from the HV to the LV terminals and vice versa.

Figure 14 shows the impedances seen from the model at the LV and HV terminals when HVA, LVA and the Case are all earthed. The LV winding presents an impedance of less than  $100 \Omega$  for most of the CENELEC A frequency band. The impedance seen when looking

into the HV terminals is very similar to the impedance of a 1 nF capacitor, for frequencies above 300 Hz.

The impedance of a typical 24 kV surge arrester is 33 pF, so that high frequency components of a lightning strike will pass through the transformer, rather than through the surge arrester prior to the surge arrester becoming a short circuit due to over voltage.

All the measurements on the transformer have been done without 50 Hz power being applied to the transformer. There may be some changes in the impedances when this is done, however the author does not expect this to be significant at CENELEC A frequencies.

## Conclusion

Modern circuit simulation and optimisation techniques can be used to develop a simple and accurate model for a power transformer. This model aids in the understanding of the behaviour of power distribution transformers when PLC signals are applied to LV or MV distribution networks for smart metering applications.

This transformer model can be used to design efficient coupling networks that couple the PLC signals onto the LV or MV lines, without the transformer absorbing all the PLC signal power. The technique described for developing the model of the power transformer can be applied to accurately model other complex devices.

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